Exercise 19
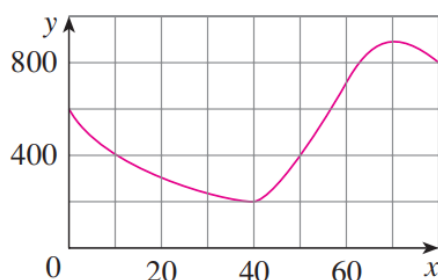
For the function f graphed in Exercise 18:

(a) Estimate the value of $f'(50)$.

(b) Is $f'(10) > f'(30)$?

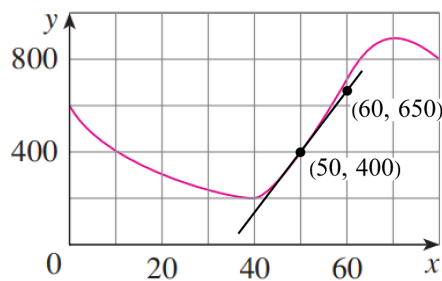(c) Is $f'(60) > \dfrac{f(80) - f(40)}{80 - 40}$? Explain.

Solution

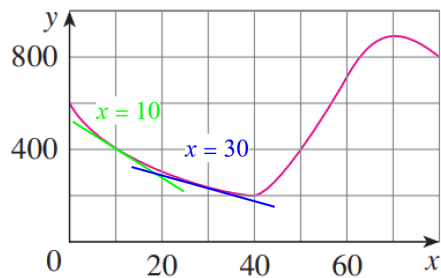The graph from Exercise 18 is shown below.

Part (a)

To estimate the value of $f'(50)$, draw the tangent line to the graph at $x = 50$ and label two points on it to calculate the slope.

$$f'(50) = m = \frac{y_2 - y_1}{x_2 - x_1} \approx \frac{650 - 400}{60 - 50} = \frac{250}{10} = 25$$

Part (b)

Draw the tangent lines to the graph at $x = 10$ and $x = 30$.
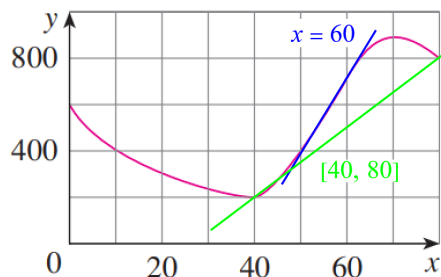

The one at $x = 10$ has a more negative slope. Therefore,

$$f'(10) < f'(30).$$

Part (c)

Draw the tangent line to the graph at $x = 60$ and draw the secant line to the graph over $[40, 80]$.

The tangent line at $x = 60$ has a higher slope than the secant line over $[40, 80]$. Therefore,

$$f'(60) > \frac{f(80) - f(40)}{80 - 40}.$$